Ray tracing and Monte Carlo methods for uncertainty quantification in the pointing direction of radio telescopes for CMB observations.

Lorenzo Liuzzo

18 Luglio 2025

Motivations for this thesis

Cosmology and The Big Bang Theory

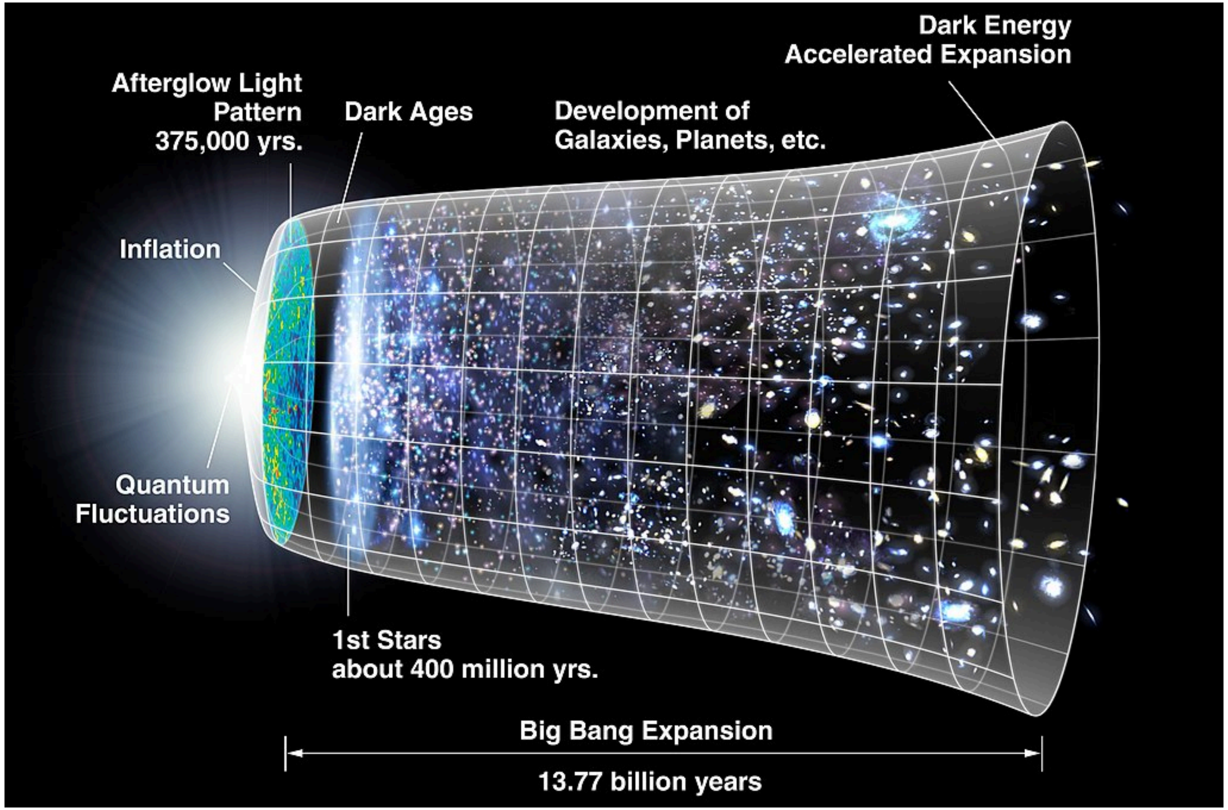

Figure 1: Evolution of the Universe according to the Big Bang theory and inflation.

Cosmic Microwave Background

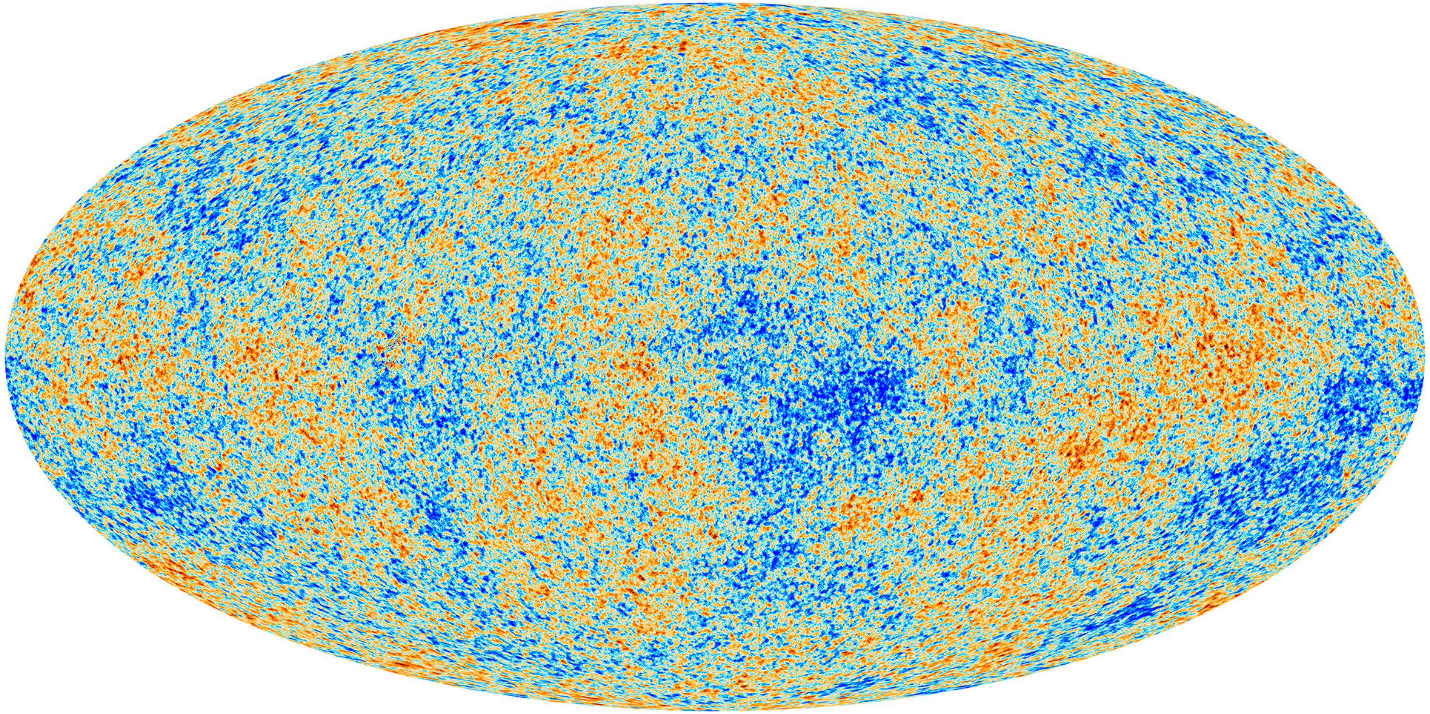

Figure 2: CMB observed by Planck instrument, image from ESA.

CMB Radio Telescopes

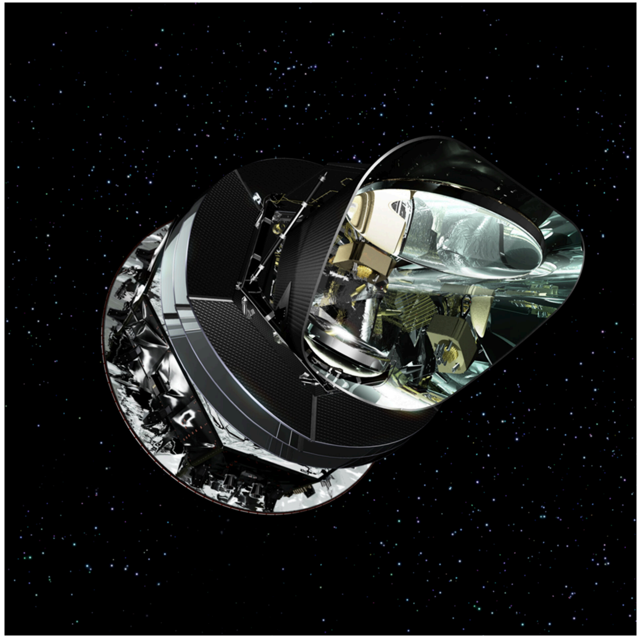

Figure 3: The Planck spacecraft, image from ESA.

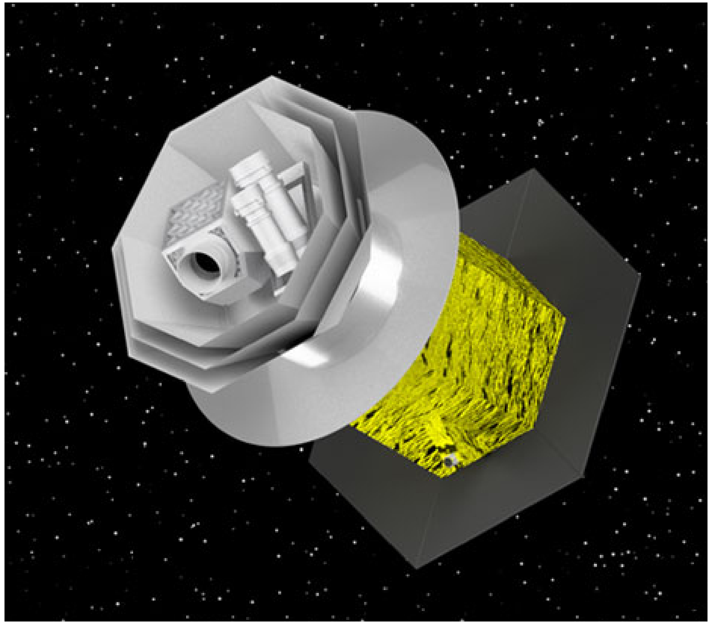

Figure 4: The LiteBIRD spacecraft, image from ISAS.

CMB Radio Telescopes

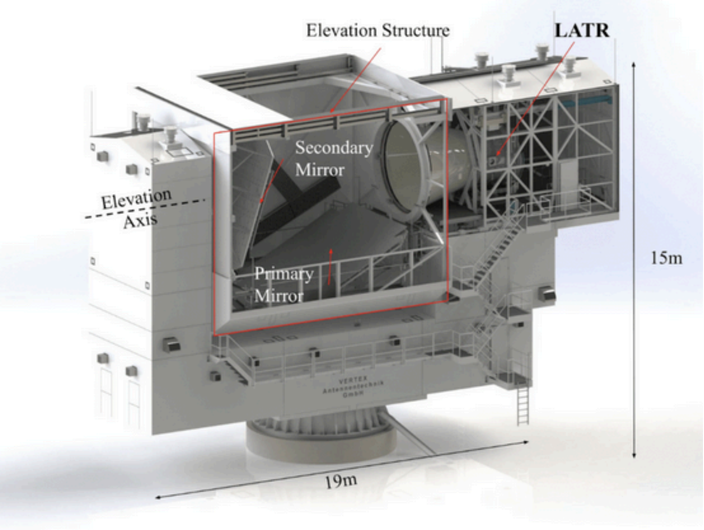

Figure 5: LATR telescope at Simons Observatory, in the Atacama Desert of Northern Chile.

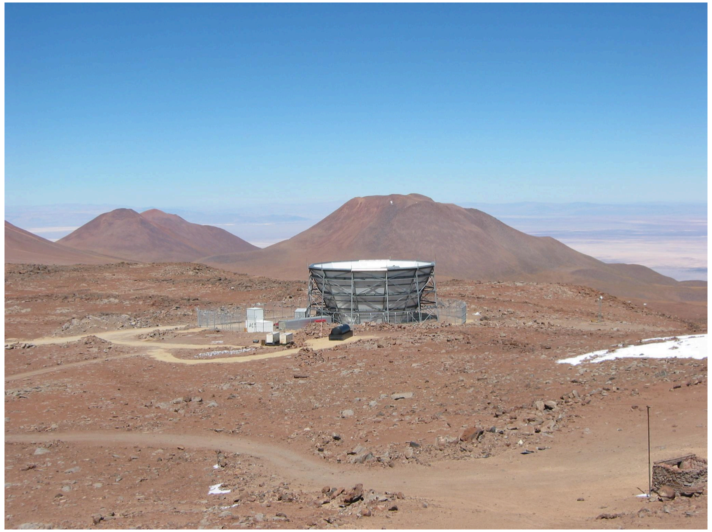

Figure 6: Atacama Cosmology Telescope (ACT), in the Atacama Desert of Northern Chile.

The Goal

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- There are already **proprietary softwares** (e.g., TICRA) that offer both ray-tracing and more accurate full-wave analysis, but they are computationally and **economically expensive**.
- In many cases we can just rely on **geometrical optics**, or often called **ray optics**.

Ray Optics

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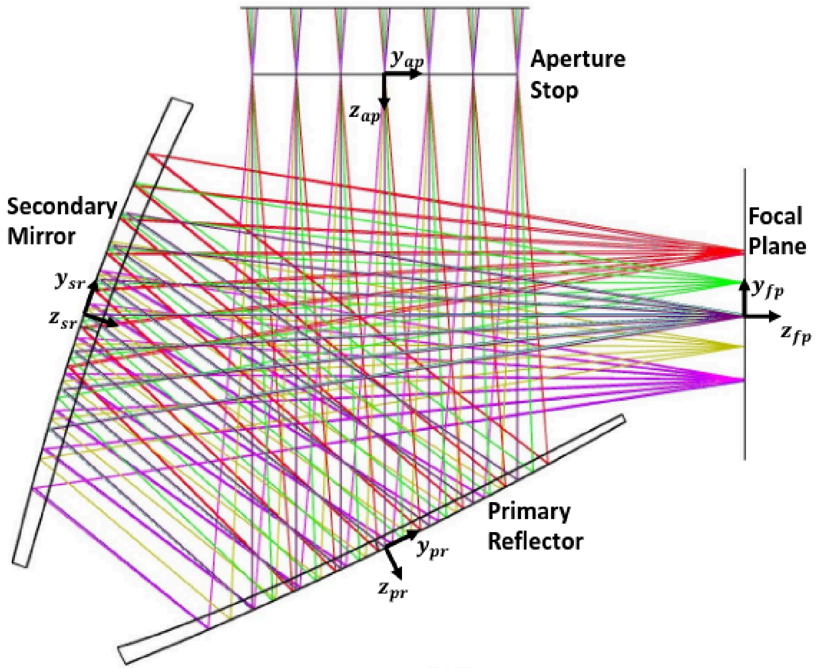
  → optical path taken by light is **reversible**
  → trace rays from the focal plane to the sky



Ray-Tracing + Monte Carlo

- Using the **Julia** programming language, we created a **software** that can model **any telescope's reflective optical system** and implements a ray-tracing algorithm to assess the **pointing direction**.

- Jointly with **Monte Carlo simulations**, it is possible to explore **many configurations** of the system under a vast range of perturbations, in order to **propagate the uncertainty** in the telescope configuration into the measurements.

The LSPE/STRIP telescope

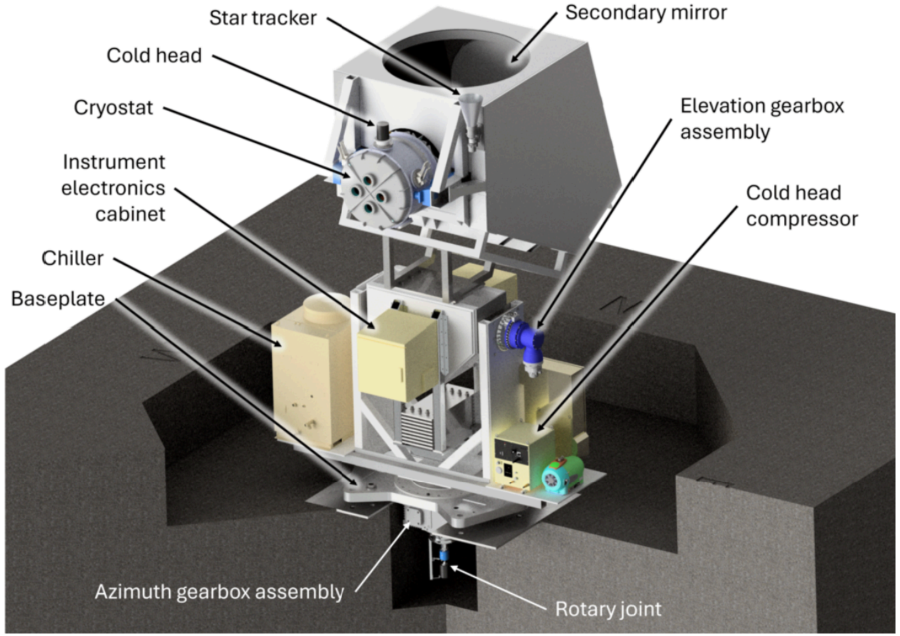


Figure 7: Rendering of LSPE-STRIP full instrument.

- As a case study, we investigated the pointing accuracy of the ***LSPE/STRIP***'s dual-reflector crossed-Dragone optical system under **tilting perturbations**.

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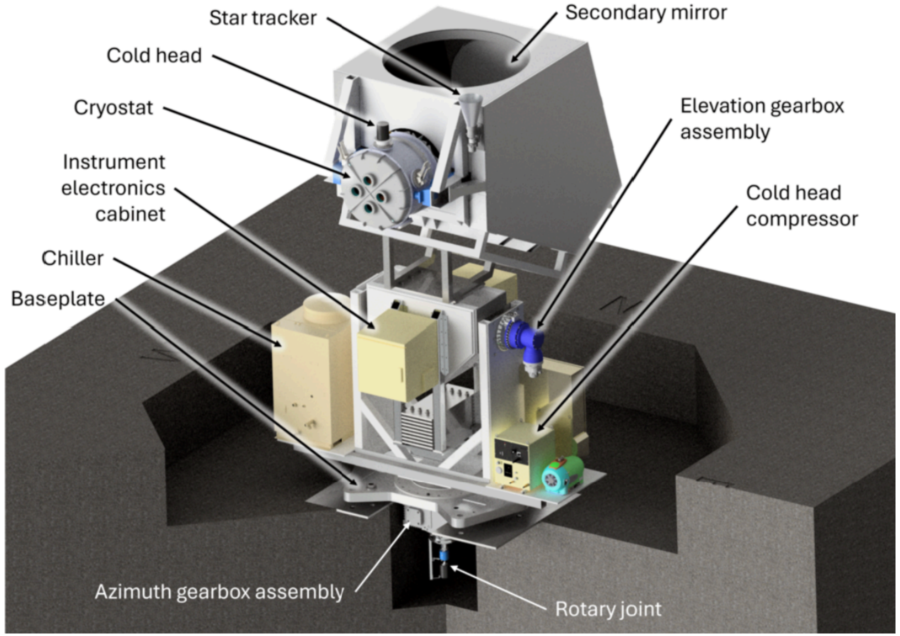

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- As a case study, we investigated the pointing accuracy of the ***LSPE/STRIP***'s dual-reflector crossed-Dragone optical system under **tilting perturbations**.
- We **integrated** the pointing direction, evaluated with the ray-tracing software, directly into the STRIP **end-to-end pipeline** to simulate sky observations.

Ray-Tracing & MC Simulations

Tilted Reflectors and Pointing Error

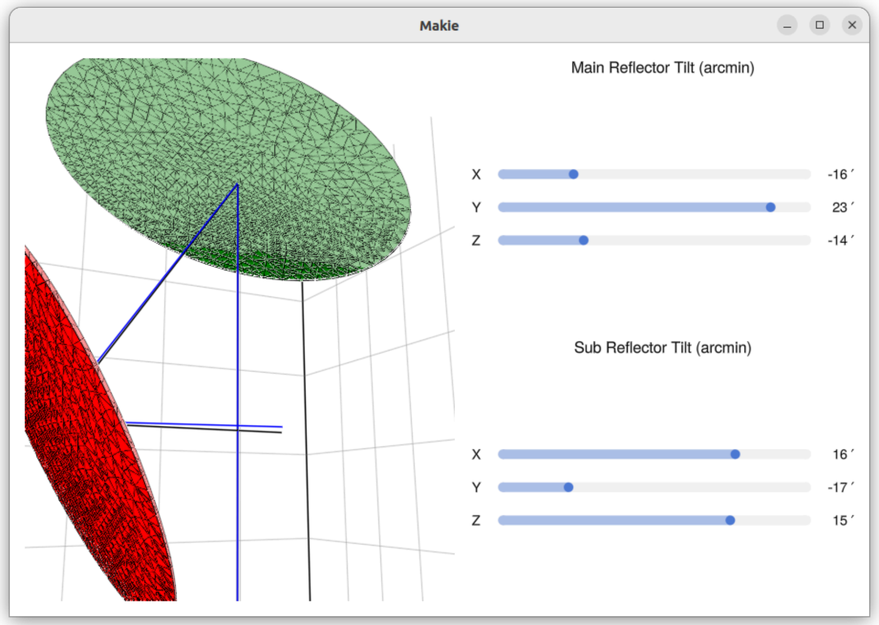

Figure 8: Interactive demo of a tilting simulation implemented with our ray-tracing engine and `Makie.jl`.

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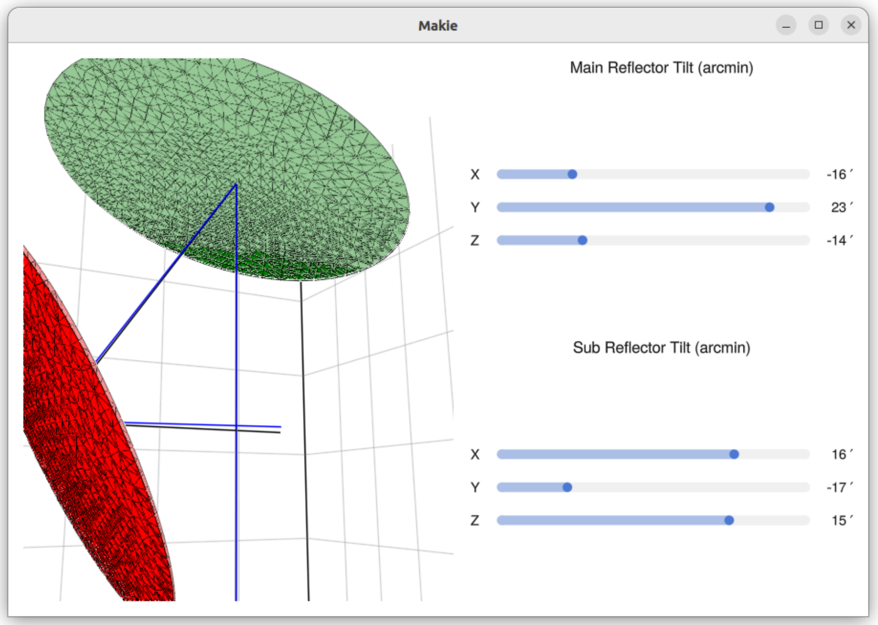

Figure 8: Interactive demo of a tilting simulation implemented with our ray-tracing engine and `Makie.jl`.

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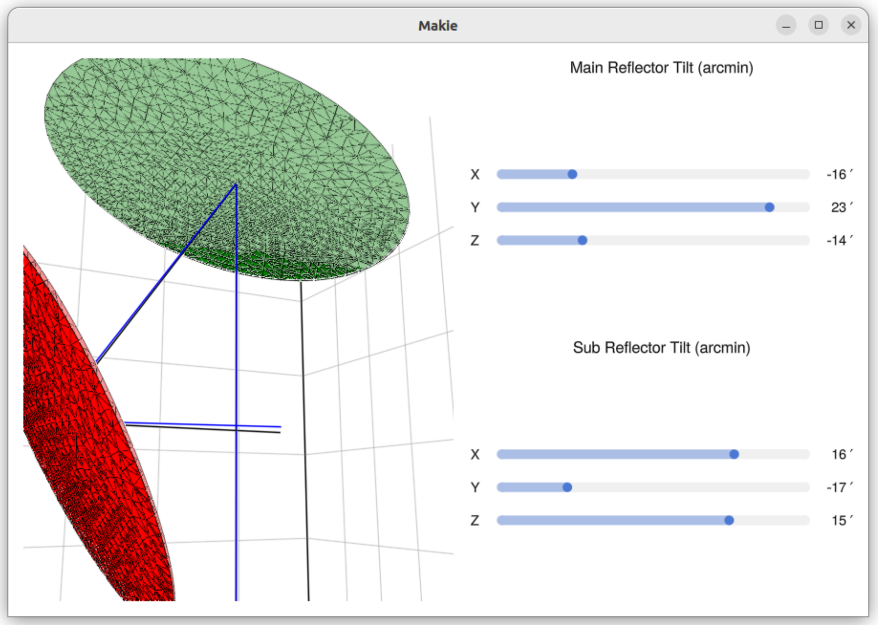

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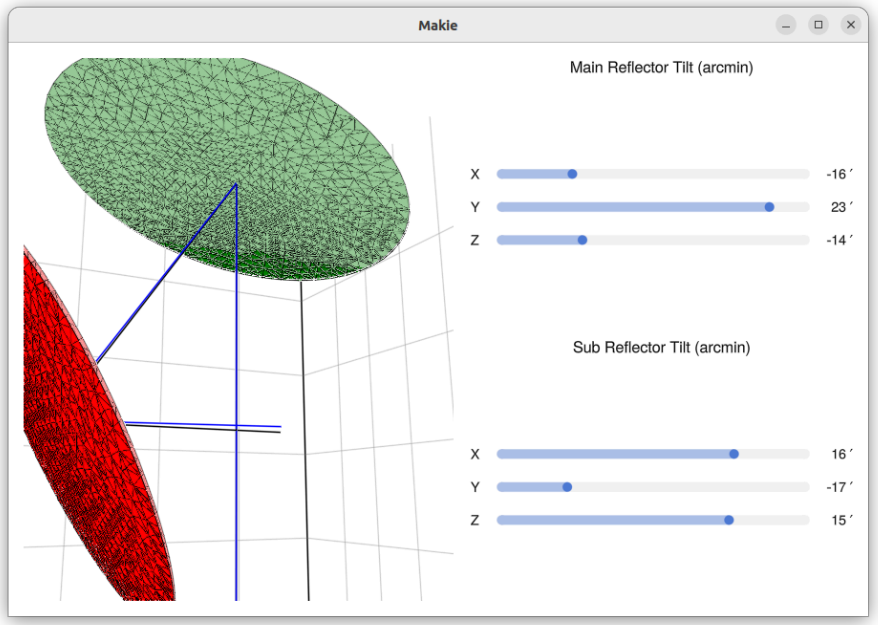

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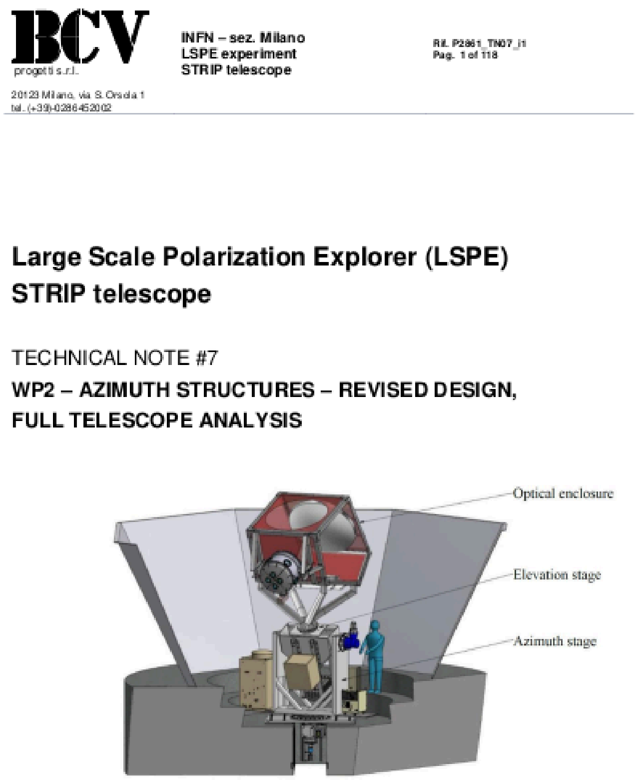
$$\text{POINTING ERROR} \equiv \arccos\left(\hat{P}_{\text{ideal}} \cdot \hat{P}_{\text{tilted}}\right)$$

Simulations SetUp


BCV progetti s.r.l.
20123 Milano, via S. Orsola 1
tel. (+39)-0286452002

INFN – sez. Milano
LSPE experiment
STRIP telescope

Rif. P2861_TN07_i1
Pag. 1 of 118

Large Scale Polarization Explorer (LSPE)
STRIP telescope

TECHNICAL NOTE #7
WP2 – AZIMUTH STRUCTURES – REVISED DESIGN, FULL TELESCOPE ANALYSIS

- We consider **independent** tilt angles

$$\theta_{a_i} \sim \mathcal{N}(0, \sigma^2)$$

$$a \in \{\text{primary}, \text{secondary}\}, \quad i \in \{\text{x}, \text{y}, \text{z}\}$$

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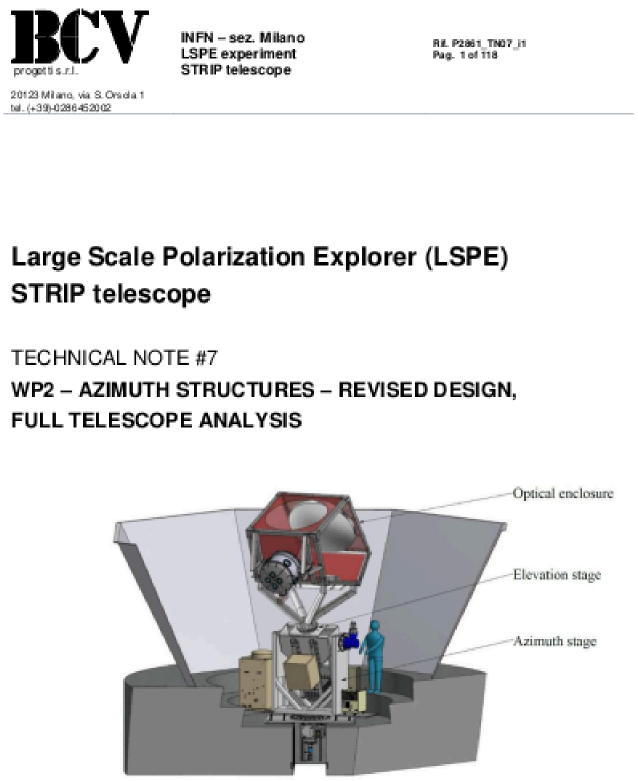

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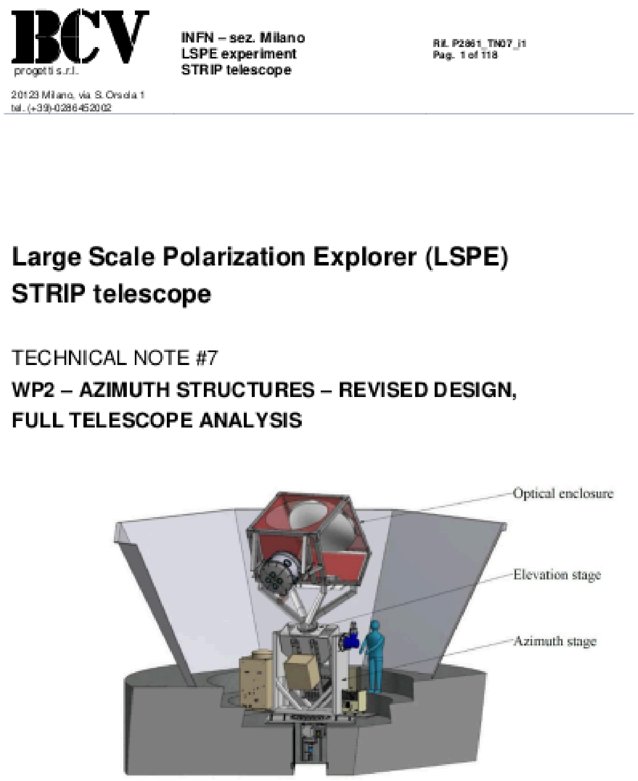

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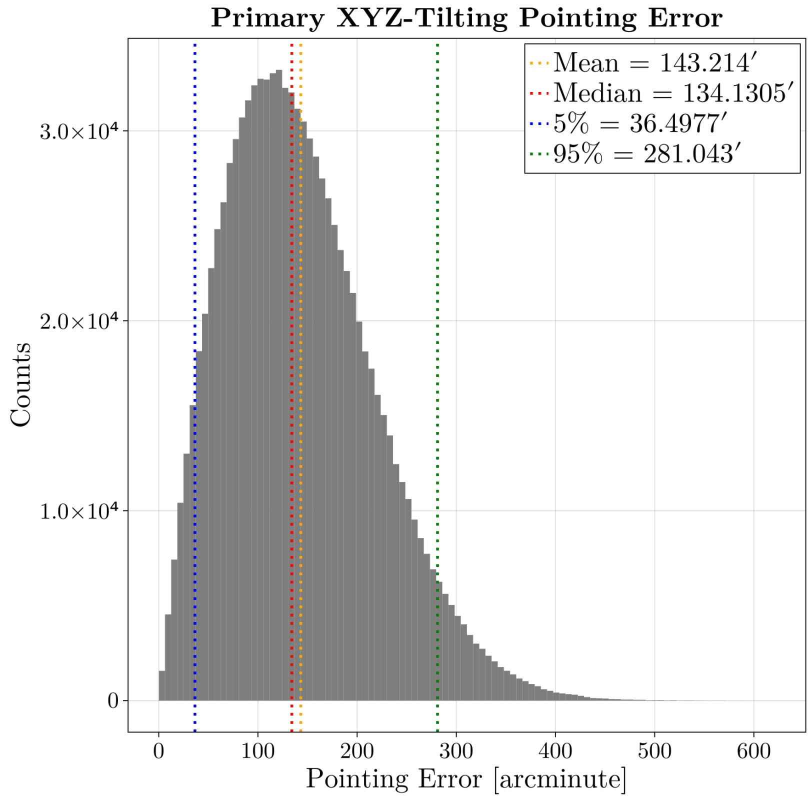
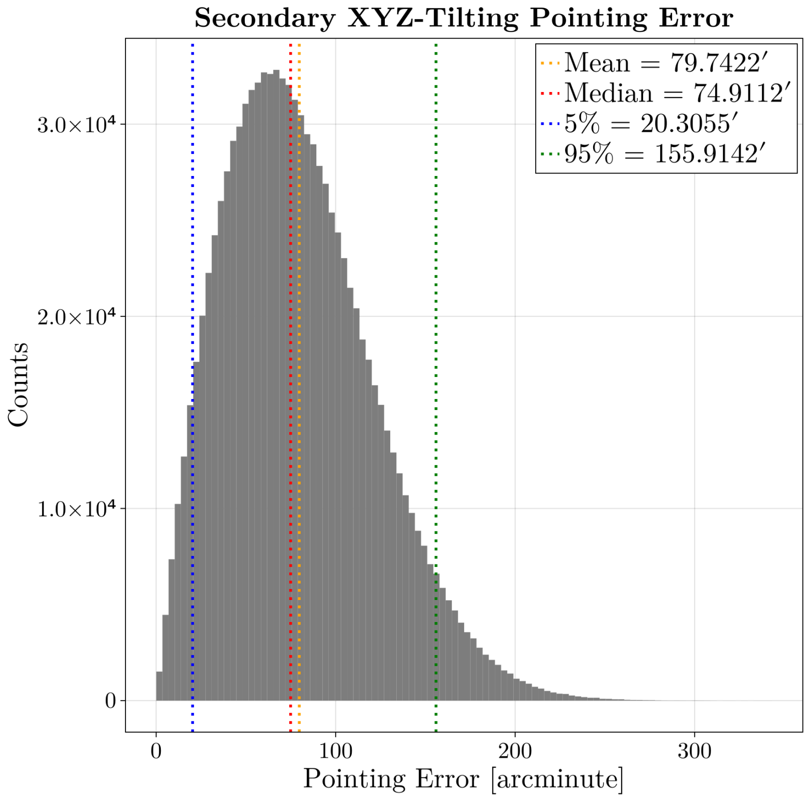
- Later we studied how the **pointing error scales** with the variance σ^2.

Multiple-Axis Single-Reflector Tilt





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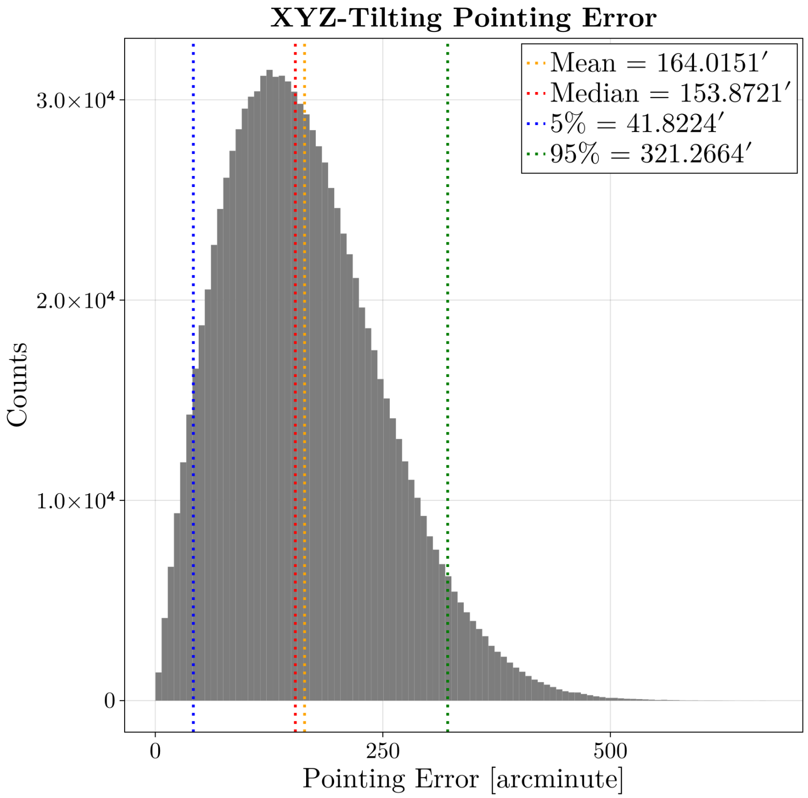
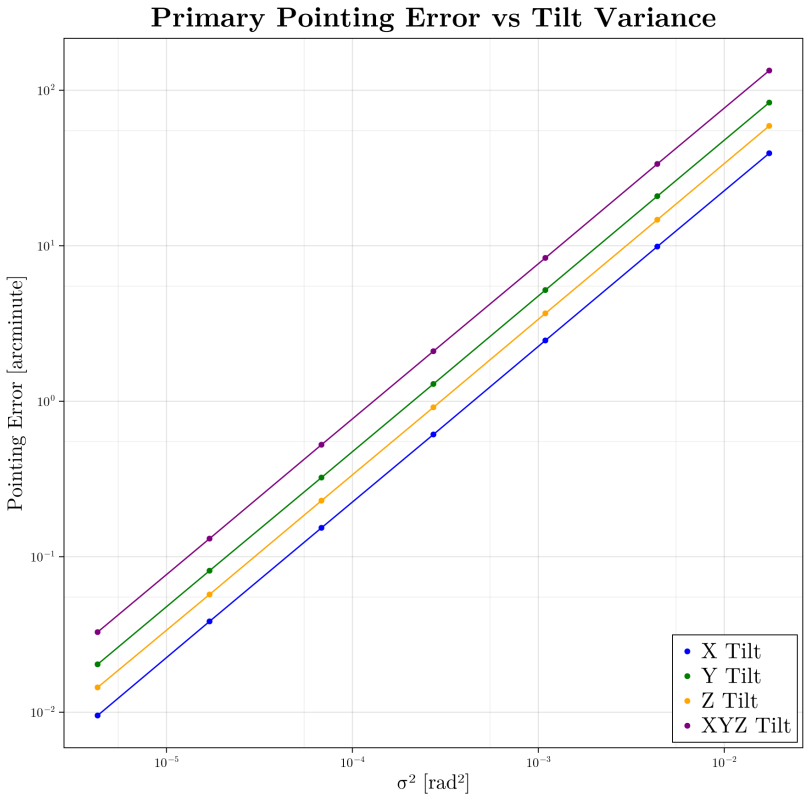
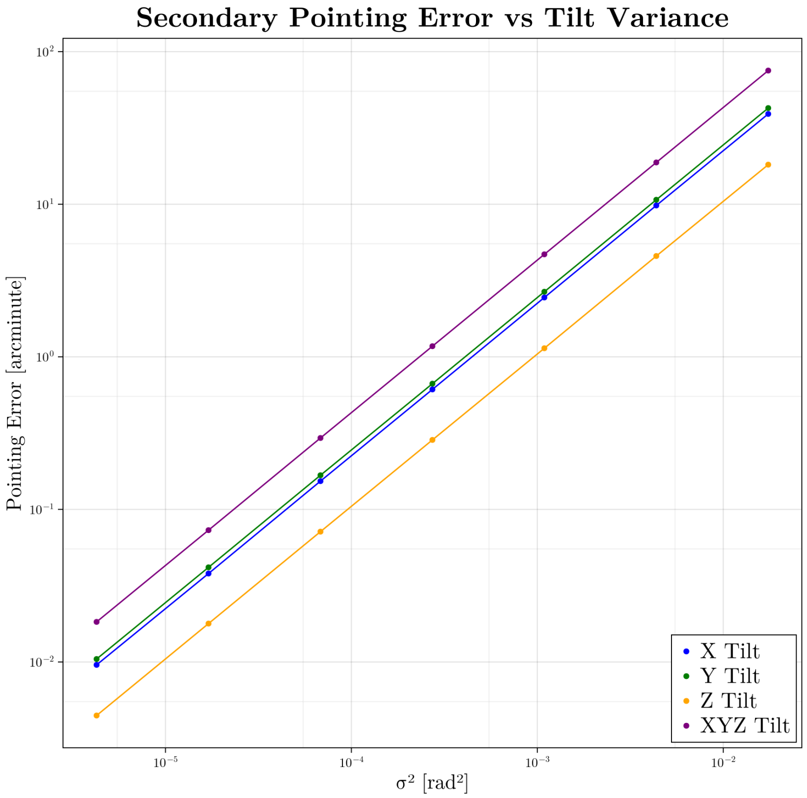


Pointing Error Trends





CMB Error Maps

Sky Observations using `STRIPLINE.jl`

We integrated the pointing direction evaluated by our ray-tracer into `STRIPLINE.jl`, the end-to-end scanning strategy pipeline for STRIP.

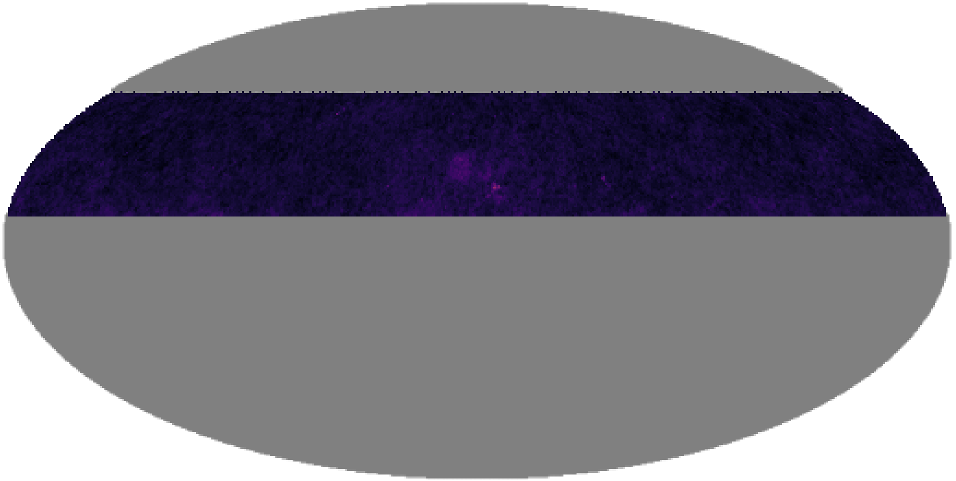


Figure 9: Simulated sky map observed by STRIP in a year of sky observation. The telescope, situated at the Canary Islands, will observe a strip of northern sky.

Sky Observations using `STRIPLINE.jl`

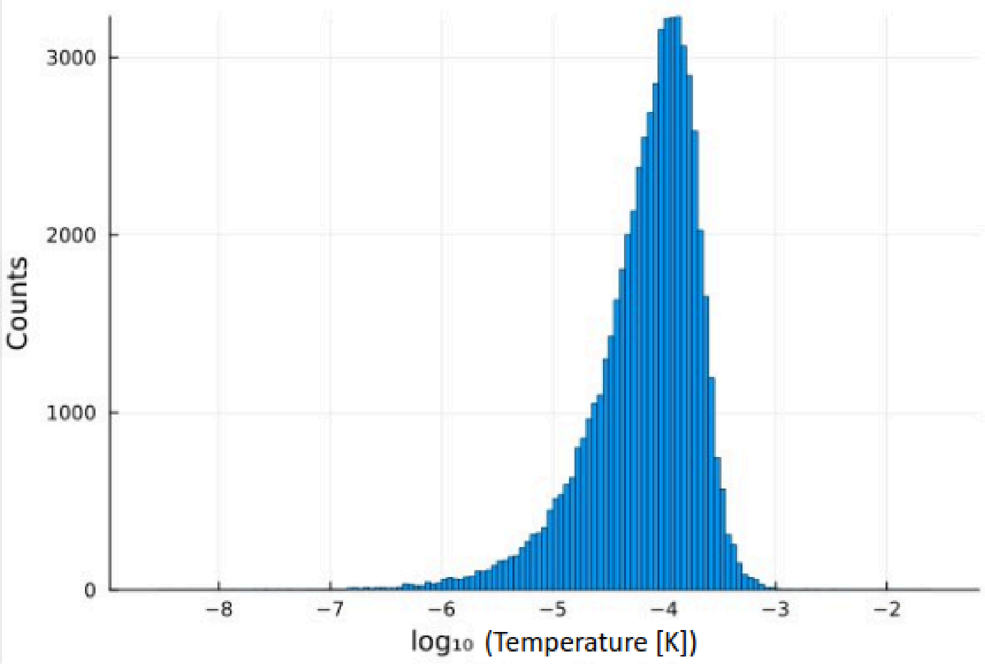


Figure 10: Histogram of the logarithm in base 10 of the error maps pixels.

Conclusion and Outlooks

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- Through the **development of a ray-tracing engine** using the Julia programming language, the application with **Monte Carlo techniques** and the **integration into** the STRIP simulation **pipeline**, this study effectively demonstrates a flexible and computationally efficient **framework for optical validation** of CMB measurements.

Conclusion

- Through the **development of a ray-tracing engine** using the Julia programming language, the application with **Monte Carlo techniques** and the **integration into** the STRIP simulation **pipeline**, this study effectively demonstrates a flexible and computationally efficient **framework for optical validation** of CMB measurements.

- To our knowledge, this represents the **first integration** of a geometrical ray-tracing engine into a full end-to-end scanning strategy pipeline.

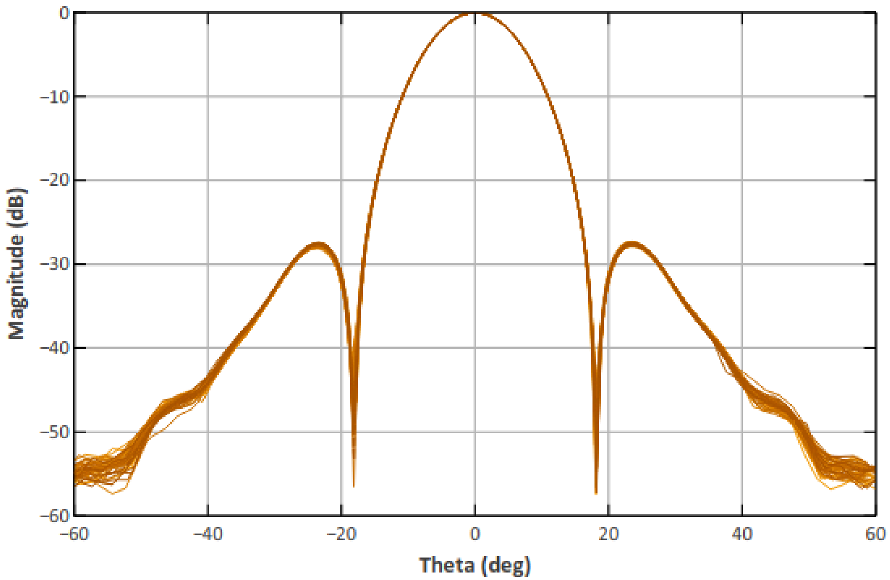
Outlooks

- Investigate **tilt angles correlations**.
- **AD** for uncertainty propagation, optimization problems and visualization of the pointing error gradients.
- Model surface imperfections with **Zernike Polynomials**.
- **Diffraction effects** with PO and MoM.
- Analyze **different kinds of perturbations** on **different telescopes**.

Thank you for the attention

Appendix

Feed’s radiation pattern



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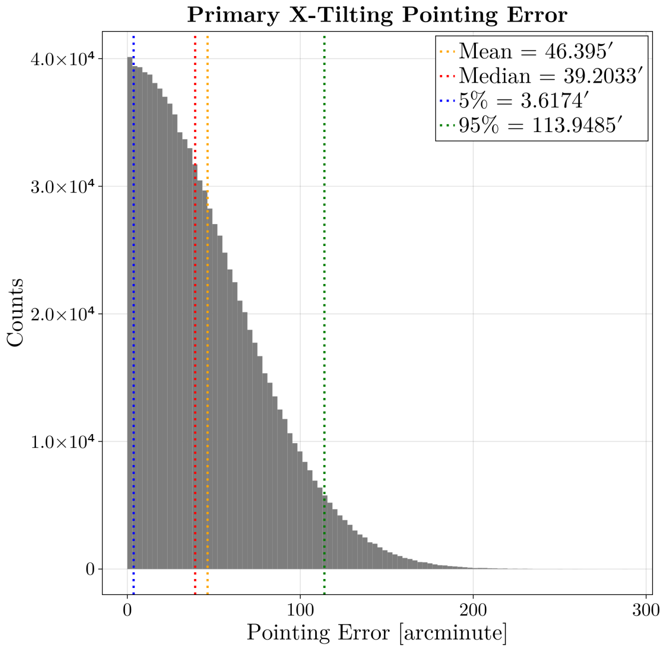

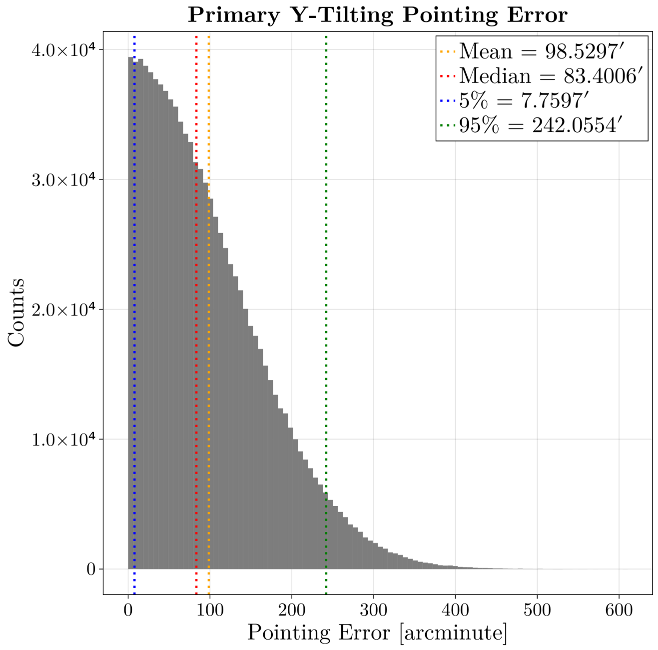

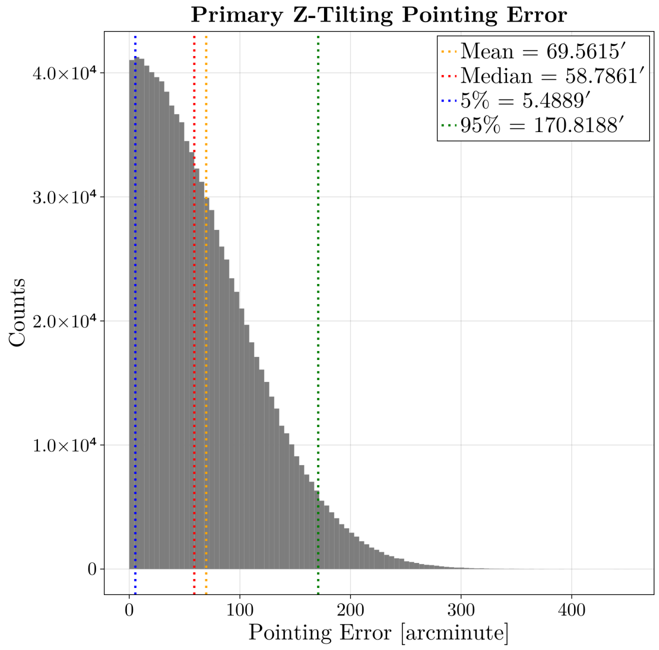

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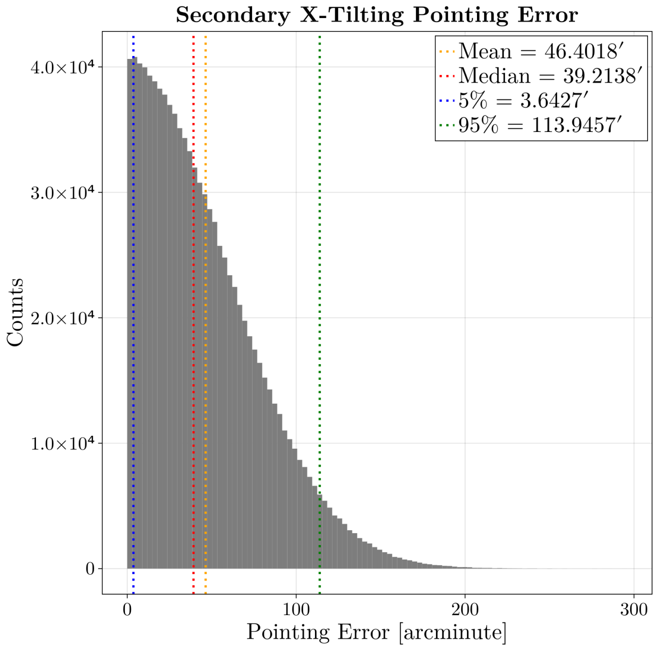

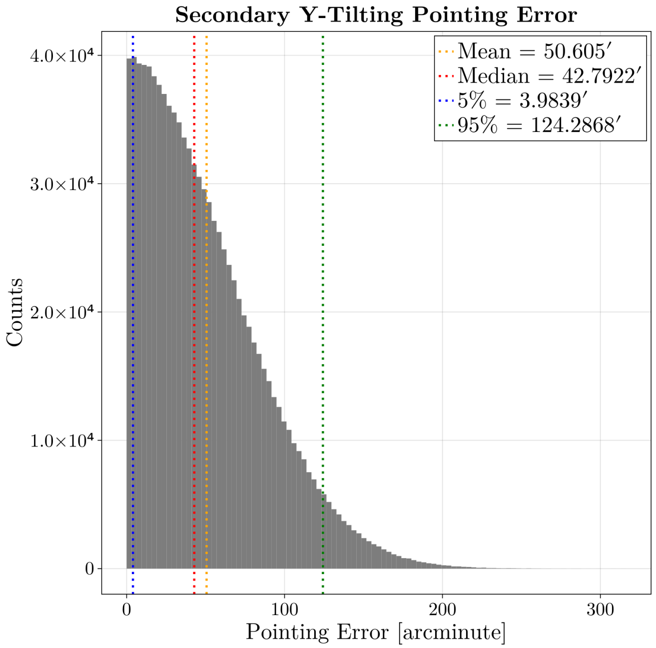

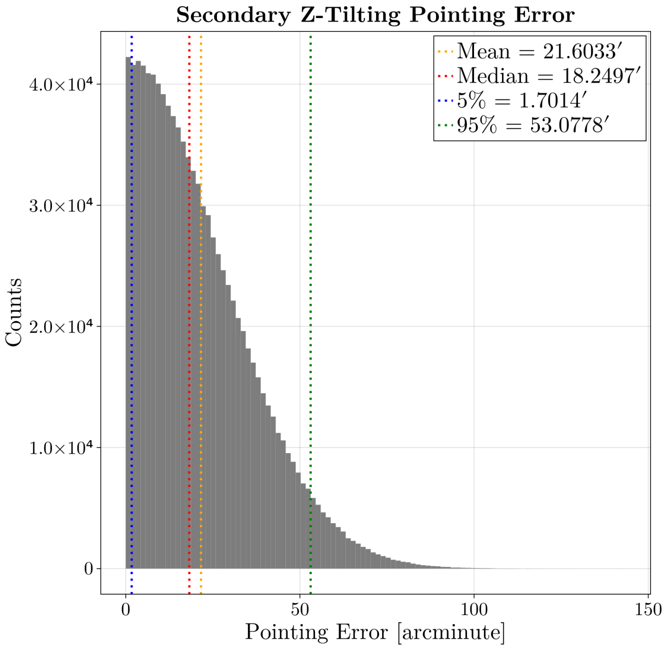

Let’s build a ray-tracing engine!

Julia Programming Language

- Modern language for **scientific** computing.
- Combines Python-like syntax with **C-like performance**.
- Excellent packages: `StaticArrays.jl`, `CoordinateTransformations.jl` and `Makie.jl`.
- Handles **units** and **automatic differentiation** very easily with type dispatch.
- An existing end-to-end simulation, `STRIPLINE.jl`, that models the **scanning strategy** of the telescope up to the creation of **frequency maps**.

Julia implementation of a Ray

Julia

```
using StaticArrays

struct Ray{O, D}
    origin::SVector{3, O}
    direction::SVector{3, D}
end

(ray::Ray)(t::Real) = ray.origin + ray.direction * t
```

Julia implementation of an AbstractSurface

```julia
abstract type AbstractSurface end

function (Σ::AbstractSurface)(u, v) = SVector(u, v, height(Σ, u, v))

function height(Σ::AbstractSurface, u, v) end
function gradient(Σ::AbstractSurface, u, v) end
function normal(Σ::AbstractSurface, u, v) end

function quad_intersection(Σ::AbstractSurface, ray) end
```

Julia implementation of a Reflector

```julia
using CoordinateTransformations

const Mapping{T} = Tuple{T, Transformation}

struct Reflector{Σ <: AbstractSurface, Γ <: AbstractRim}
    surface::Mapping{Σ}
    rim::Mapping{Γ}
    name::String
end
```